Phase Angle and Phase Shift

**EXERCISE OBJECTIVE**
When you have completed this exercise, you will know what a phase angle is and how the phase angle modifies the initial displacement of a sine wave. You will be able to determine the phase shift between two sine waves, either by comparing their phase angles or by determining their separation in time. You will also know how to distinguish a leading phase shift from a lagging phase shift.

**DISCUSSION OUTLINE**
The Discussion of this exercise covers the following points:
- Phase angle
- Phase shift

**DISCUSSION**

**Phase angle**

As you have seen in Exercise 1-1, the graphical representation of a sine wave can be expressed by the following equation:

\[ a(t) = A \sin(\omega t) \]  

(1-8)

where \( a(t) \) is the instantaneous value of the sine wave at a given instant \( t \).
- \( A \) is the amplitude of the sine wave.
- \( \omega \) is the angular velocity, expressed in radians per second (rad/s).
- \( t \) is the time, expressed in seconds (s).

This equation assumes that the sine wave cycle begins at the exact moment when \( t = 0 \) (as is shown in Figure 1-10). As you will see later, this is not always the case. To represent the initial position of the sine wave, the notion of **phase angle** \( \theta \) is introduced in the equation below:

\[ a(t) = A \sin(\omega t + \theta) \]  

(1-9)

where \( \theta \) is the phase angle of the sine wave, expressed in degrees (°) or radians (rad).

From Equation (1-9), it is easy to observe that the initial value (i.e., the value at \( t = 0 \)) of the sine wave depends entirely on the phase angle \( \theta \) because the term \( \omega t \) equals 0 at \( t = 0 \). In other words, the phase angle \( \theta \) determines by how much the value of a sine wave differs from 0 at time \( t = 0 \), and thus, the position in time of the sine wave.
Figure 1-10 shows a sine wave with a phase angle \( \theta \) of 0°. The initial value of this sine wave is 0 because \( A \sin(\theta \cdot 0 + 0) = 0 \). This sine wave is identical to those seen in Exercise 1-1, as a phase angle value of 0° was implied by the absence of \( \theta \) in the equations given in Exercise 1-1.

![Figure 1-10. Sine wave with a phase angle \( \theta \) of 0°.](image)

Figure 1-11 shows a sine wave with a phase angle \( \theta \) of 45°. As you can see from the figure, a positive phase angle (0° to 180°) results in the sine wave having a positive instantaneous value when \( t = 0 \). In other words, a positive phase angle shifts the sine wave toward the left, i.e., advances the sine wave in time.

![Figure 1-11. Sine wave with a phase angle \( \theta \) of 45°.](image)

Figure 1-12 shows a sine wave with a phase angle \( \theta \) of -60°. A negative phase angle (0° to -180°) results in the sine wave having a negative instantaneous value when \( t = 0 \). In other words, a negative phase angle shifts the sine wave toward the right, i.e., delays the sine wave in time.

![Figure 1-12. Sine wave with a phase angle \( \theta \) of -60°.](image)
Figure 1-10 to Figure 1-12 also show the phasor representations of the sine waves at time $t = 0$. Notice that, in each figure, the vertical distance between the tip of the rotating phasor representing the sine wave matches the instantaneous value of the sine wave at $t = 0$.

**Phase shift**

When comparing two sine waves having the same frequency, the difference between their respective phase angles is called the phase shift and is expressed in degrees (°) or radians (rad). The magnitude of the phase shift indicates the extent of separation in time between the two sine waves, while the polarity of the phase shift (positive or negative) indicates the relationship in time between the two sine waves (leading or lagging). The sine wave amplitude value has no effect on the phase shift, as it does not change the period nor the frequency of the sine wave. Sine waves with different frequencies and, as an extension, different periods, cannot be compared by using their phase angles as their cycles do not correspond.

The phase shift between two sine waves is expressed as an angle representing a portion of a complete cycle of the sine waves. One of the two sine waves is used as the reference for phase shift measurements. The phase shift is calculated by subtracting the phase angle $\theta_{\text{Ref.}}$ of the reference sine wave from the phase angle $\theta$ of the sine wave of interest. This is written as an equation below.

$$\text{Phase shift} = \theta - \theta_{\text{Ref.}} \quad (1-10)$$

where $\theta$ is the phase angle of the sine wave of interest, expressed in degrees (°) or radians (rad).

$\theta_{\text{Ref.}}$ is the phase angle of the reference sine wave, expressed in degrees (°) or radians (rad).

Figure 1-13 is an example showing how the phase shift between two sine waves (X and Y) can be calculated using their phase angles.

In the figure, sine wave X has a phase angle $\theta$ of 45°, while sine wave Y has a phase angle $\theta$ of -60°. Depending on which sine wave is used as the reference, the phase shift can be +105° or -105°. When sine wave X is considered as the reference, the phase shift of sine wave Y with respect to sine wave X is -105°.
(-60° - 45° = -105°). The minus sign in this phase shift value indicates that sine wave Y lags reference sine wave X. For this reason, this phase shift value can also be expressed as 105° lagging. Conversely, when sine wave Y is considered as the reference, the phase shift of sine wave X with respect to sine wave Y is +105° (45° - (-60)° = +105°). The plus sign in this phase shift value indicates that sine wave X leads reference sine wave Y. For this reason, this phase shift value can also be expressed as 105° leading. Note that whenever two sine waves have different phase angles, the phase shift value is not zero, and thus, these sine waves are said to be out of phase.

It is possible to determine the phase shift between two sine waves of the same frequency without knowing their respective phase angles \( \theta \). The following equation is used:

\[
\text{Phase shift} = \frac{d}{T} \times 360° = \frac{d}{T} \times 2\pi \text{ rad} \quad (1-11)
\]

where \( d \) is the time interval between a given reference point on each of the two sine waves, expressed in seconds (s).

\( T \) is the period of the sine waves, expressed in seconds (s).

This equation shows in a concrete way why it is not possible to calculate the phase shift between two sine waves having different frequencies \( f \), as a common period \( T (T = 1/f) \) is needed for the equation to be valid.

Consider, for example, the sine waves shown in Figure 1-14. Using Equation (1-11), the phase shift between the two sine waves is equal to:

\[
\text{Phase shift} = \frac{d}{T} \times 360° = \frac{3.33 \text{ ms}}{20.0 \text{ ms}} \times 360° = 60°
\]

When sine wave 1 is used as the reference, the phase shift is lagging because sine wave 2 is delayed with respect to sine wave 1. Conversely, when sine wave 2 is considered as the reference, the phase shift is leading because sine wave 1 is in advance with respect to sine wave 2.
Figure 1-14. Phase shift between two sine waves having the same frequency.

**PROCEDURE OUTLINE**

The Procedure is divided into the following sections:

- Setup and connections
- Measuring the phase shift between two voltage sine waves in a resistor-inductor (RL) circuit
- Measuring the phase shift between two voltage sine waves in a resistor-capacitor (RC) circuit

**PROCEDURE**

High voltages are present in this laboratory exercise. Do not make or modify any banana jack connections with the power on unless otherwise specified.

**Setup and connections**

*In this section, you will connect an ac circuit containing an inductor and a resistor in series and set up the equipment to measure the source voltage $E_s$, as well as the voltage across the resistor $E_R$.*

1. Refer to the Equipment Utilization Chart in Appendix A to obtain the list of equipment required to perform this exercise.

   Install the required equipment in the **Workstation**.
2. Make sure that the main power switch on the Four-Quadrant Dynamometer/Power Supply is set to the O (off) position, then connect its Power Input to an ac power outlet.

   Connect the Power Input of the Data Acquisition and Control Interface to a 24 V ac power supply. Turn the 24 V ac power supply on.

3. Connect the USB port of the Data Acquisition and Control Interface to a USB port of the host computer.

   Connect the USB port of the Four-Quadrant Dynamometer/Power Supply to a USB port of the host computer.

4. Turn the Four-Quadrant Dynamometer/Power Supply on, then set the Operating Mode switch to Power Supply.

5. Turn the host computer on, then start the LVDAC-EMS software.

   In the LVDAC-EMS Start-Up window, make sure that the Data Acquisition and Control Interface and the Four-Quadrant Dynamometer/Power Supply are detected. Make sure that the Computer-Based Instrumentation function for the Data Acquisition and Control Interface is available. Select the network voltage and frequency that correspond to the voltage and frequency of your local ac power network, then click the OK button to close the LVDAC-EMS Start-Up window.

6. Set up the circuit shown in Figure 1-15. This circuit contains a resistor $R$ and an inductor $L$. Inductors are studied in the next unit of this manual.

   ![AC circuit with a resistor and an inductor](image)

   Figure 1-15. AC circuit with a resistor and an inductor.

   The value of inductor $L$ in the circuit of Figure 1-15 is referred to as the inductance and is expressed in henries (H). The inductance value to be used depends on the frequency of the ac power source as is indicated in Table 1-2.
Procedure

As indicated in Appendix A, use the Inductive Load, Model 8321, to obtain the required inductance when the ac power network frequency is 60 Hz. Use the Inductive and Capacitive Loads, Model 8333, to obtain the required inductance when the ac power network frequency is 50 Hz.

<table>
<thead>
<tr>
<th>Power source frequency (Hz)</th>
<th>Inductance (H)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>0.96</td>
</tr>
<tr>
<td>60</td>
<td>0.8</td>
</tr>
</tbody>
</table>

Make the necessary switch settings on the Resistive Load and Inductive Load (or on the Inductive and Capacitive Loads) in order to obtain the resistance and inductance values required.

Appendix C of this manual lists the switch settings to implement on the Resistive Load and Inductive Load in order to obtain various resistance and inductive values.

Use inputs $E_1$ and $E_2$ of the Data Acquisition and Control Interface to measure the source voltage $E_S$ and the voltage across the resistor $E_R$, respectively.

7. In LVDAC-EMS, open the Four-Quadrant Dynamometer/Power Supply window, then make the following settings:
   - Set the Function parameter to AC Power Source.
   - Make sure that the Voltage Control parameter is set to Knob. This allows the ac power source to be controlled manually.
   - Set the No Load Voltage parameter to 100 V.
   - Set the Frequency parameter to the frequency of your local ac power network.
   - Leave the other parameters set as they are.

Measuring the phase shift between two voltage sine waves in a resistor-inductor (RL) circuit

In this section, you will observe the waveforms (sine waves) of the source voltage $E_S$ and the resistor voltage $E_R$, using the Oscilloscope to determine the phase shift between the two sine wave voltages. Then, using the Phasor Analyzer, you will measure the phase shift between the source voltage phasor and the resistor voltage phasor and compare it to the phase shift determined from the voltage waveforms.

As you will see later, due to the presence of an inductor in the circuit, the circuit current lags behind the source voltage. As a result, the voltage $E_R$ measured across the resistor is out of phase with respect to the source voltage $E_S$. 
8. In LVDAC-EMS, open the Metering window. Set meters \( E1 \) and \( E2 \) to measure the rms values of the source voltage \( E_s \) and voltage across the resistor \( E_R \), respectively.

In the Four-Quadrant Dynamometer/Power Supply window, enable the ac power source. Readjust the value of the No Load Voltage parameter so that the ac power source voltage \( E_s \) (indicated by meter \( E1 \) in the Metering window) is equal to 100 V.

9. In LVDAC-EMS, open the Oscilloscope and display \( E_s \) (input \( E1 \)) and \( E_R \) (input \( E2 \)) on channels 1 and 2, respectively. If necessary, set the time base so as to display at least two cycles of the sine waves. Place the traces of the two channels at the same vertical position.

10. Measure the period \( T \) of the source voltage \( E_s \) using the Oscilloscope then record the value below.

   To obtain an accurate measurement, you can use the vertical cursors of the Oscilloscope to measure the period or any other time interval.

   Period \( T = \) _______ ms

11. Measure the period \( T \) of the resistor voltage \( E_R \) using the Oscilloscope then record the value below.

   Period \( T = \) _______ ms

12. Compare the period \( T \) of the resistor voltage \( E_R \) measured in the previous step with the period \( T \) of the source voltage \( E_s \) recorded in step 10. Are the values close to each other?

   - [ ] Yes
   - [ ] No

13. Measure the time interval \( d \) between the waveforms of the source voltage \( E_s \) and resistor voltage \( E_R \) by using the Oscilloscope.

   Time interval \( d = \) _______ ms

14. Using Equation (1-11), calculate the phase shift between the source voltage \( E_s \) and the resistor voltage \( E_R \). Consider the source voltage waveform as the reference.

   Phase shift = _______°
15. Is the resistor voltage \( E_R \) leading or lagging the source voltage \( E_S \)?

16. In LVDAC-EMS, open the Phasor Analyzer and display the source voltage \( E_S \) (input \( E1 \)) and resistor voltage \( E_R \) (input \( E2 \)). Set the Reference Phasor parameter to \( E1 \). Measure the phase angles \( \theta_{ES} \) and \( \theta_{ER} \) of the voltage phasors.

\[
\text{Phase angle } \theta_{ES} = \underline{\text{}} \text{°}
\]

\[
\text{Phase angle } \theta_{ER} = \underline{\text{}} \text{°}
\]

From these values, calculate the phase shift between the phasors of the source voltage \( E_S \) and resistor voltage \( E_R \), using the source voltage phasor as the reference.

\[
\text{Phase shift} = \underline{\text{}} \text{°}
\]

17. Compare the phase shift you determined from the voltage sine waves to the phase shift you measured from the corresponding voltage phasors. Are both values close to each other?

- [ ] Yes
- [ ] No

Measuring the phase shift between two voltage sine waves in a resistor-capacitor (RC) circuit

In this section, you will replace the inductor used in the previous section by a capacitor. Using the Oscilloscope, you will determine the phase shift between the two voltage sine waves. Then, using the Phasor Analyzer, you will measure the phase shift between the source voltage phasor and the resistor voltage phasor and compare it to the phase shift you determined from the voltage waveforms.

As you will see later, due to the presence of a capacitor in the circuit, the circuit current leads the source voltage. As a result, the resistor voltage \( E_R \) is out of phase with respect to the source voltage \( E_S \).

18. In the Four-Quadrant Dynamometer/Power Supply window, disable the ac power source.
19. Modify the circuit so that it is as shown in Figure 1-16 (replace the inductor by a capacitor). This circuit contains a resistor $R$ and a capacitor $C$. Capacitors are studied in the next unit of this manual.

![Figure 1-16. AC circuit with a resistor and a capacitor.]

The value of capacitor $C$ in the circuit of Figure 1-16 is referred to as the capacitance and is expressed in microfarads ($\mu$F). The capacitance value to be used depends on the frequency of the ac power source as is indicated in Table 1-3.

As indicated in Appendix A, use the Capacitive Load, Model 8331, to obtain the required capacitance when the ac power network frequency is 60 Hz. Use the Inductive and Capacitive Loads, Model 8333, to obtain the required capacitance when the ac power network frequency is 50 Hz.

<table>
<thead>
<tr>
<th>Power source frequency (Hz)</th>
<th>Capacitance ($\mu$F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>5.3</td>
</tr>
<tr>
<td>60</td>
<td>4.4</td>
</tr>
</tbody>
</table>

Make the necessary switch settings on the Resistive Load and Capacitive Load (or on the Inductive and Capacitive Loads) in order to obtain the resistance and capacitance values required.

20. In the Four-Quadrant Dynamometer/Power Supply window, enable the ac power source. In the Four-Quadrant Dynamometer/Power Supply window, readjust the value of the No Load Voltage parameter, if necessary, so that the ac power source voltage $E_s$ (indicated by meter $E_{1}$ in the Metering window) is equal to 100 V.

21. Measure the period $T$ of the source voltage using the Oscilloscope then record the value below.

Period $T = \underline{\hspace{2cm}}$ ms
22. Measure the period $T$ of the resistor voltage $E_R$ using the Oscilloscope then record the value below.

   Period $T = \underline{\hspace{2cm}}$ ms

23. Compare the period $T$ of the resistor voltage $E_R$ measured in the previous step with the period $T'$ of the source voltage $E_S$ recorded in step 21. Are the values close to each other?

   - Yes
   - No

24. Measure the time interval $d$ between the waveforms of the source voltage $E_S$ and resistor voltage $E_R$.

   Time interval $d = \underline{\hspace{2cm}}$ ms

25. Using Equation (1-11), calculate the phase shift between the source voltage $E_S$ and the resistor voltage $E_R$. Consider the source voltage waveform as the reference.

   Phase shift = \underline{\hspace{2cm}}^\circ

26. Is the resistor voltage $E_R$ leading or lagging the source voltage $E_S$?

27. In the Phasor Analyzer, measure the phase angles $\theta_{ES}$ and $\theta_{ER}$ of the voltage phasors.

   Phase angle $\theta_{ES} = \underline{\hspace{2cm}}^\circ$

   Phase angle $\theta_{ER} = \underline{\hspace{2cm}}^\circ$

   From these values, calculate the phase shift between the phasors of the source voltage $E_S$ and resistor voltage $E_R$, using the source voltage phasor as the reference.

   Phase shift = \underline{\hspace{2cm}}^\circ

28. Compare the phase shift you determined from the voltage sine waves to the phase shift you measured from the corresponding voltage phasors. Are both values close to each other?

   - Yes
   - No

29. In the Four-Quadrant Dynamometer/Power Supply window, disable the ac power source.
30. Close LVDAC-EMS, then turn off all the equipment. Disconnect all leads and return them to their storage location.

**CONCLUSION**

In this exercise, you saw how the phase angle modifies the value of a sine wave at time $t = 0$, and thus, the position in time of the sine wave. You observed the effects of positive and negative phase angles on the relative position in time of a sine wave. You were introduced to the notion of phase shift. You learned how to calculate and measure the phase shift between two sine waves and to differentiate between a lagging and a leading phase shift.

**REVIEW QUESTIONS**

1. What is the effect of the phase angle on the graphical representation of a sine wave?

2. A sine wave has a phase angle $\theta$ of $72^\circ$. Will this sine wave reach its maximum value before, after or at the same time as a second waveform having a phase angle $\theta$ of $-18^\circ$?

3. Given the following two sine wave equations:

   $$E(t) = 8 \sin 20t + 78^\circ$$

   $$E(t) = 40 \sin 20t + 43^\circ$$

   Calculate the phase shift between these two sine waves, considering the first sine wave as the reference. Indicate also whether the second sine wave is lagging or leading the reference sine wave.
4. When calculating the phase shift between two sine waves, which of the following parameters do the two sine waves need to have in common: phase angle, amplitude, frequency, or period? Why?

5. Consider two sine waves with the same frequency. They both have a period $T$ of 50 ms. The second sine wave reaches its maximum positive value 8 ms after the first. Calculate the phase shift between the two sine waves, considering the first one as the reference.